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Design of Maximum Thrust Nozzle Contours by Direct Optimization Methods

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Abstract

A PROCEDURE for the design of maximum thrust nozzle contours by direct optimization methods is presented. The nozzle contour is a second-degree polynomial having a fixed initial expansion contour. The coefficients of the polynomial are varied by direct optimization methods to determine the maximum thrust contour. Three direct optimization methods are considered: multidimensional line search, the method of steepest descent, and Newton's method. Results are presented to illustrate the behavior of the direct optimization methods, and to demonstrate that second-degree polynomial contours yield thrusts comparable to the thrusts developed by nozzle contours determined by calculus of variations methods.

Contents

Some procedures[1] for designing maximum thrust nozzle contours are based on the calculus of variations. A major drawback to this procedure is that if the nozzle configuration or the gasdynamic model are changed, then the entire optimization analysis and corresponding computer program must be reworked.

The objective of the present investigation[2] was to develop an efficient method for the design of maximum thrust nozzle contours by direct optimization methods. That was accomplished by specifying the nozzle contour as a polynomial, and employing nonlinear programming methods[3] to determine the polynomial that yields the maximum thrust contour for a given nozzle configuration, a given gasdynamic model, and a given design constraint.

Two questions are answered by this investigation. First, how do direct optimization methods behave in this problem? Second, do polynomial nozzle contours develop thrusts comparable to calculus of variations contours?

In the present investigation, the nozzle configuration is a conventional De Laval nozzle specified by a second-degree polynomial, the gasdynamic model assumes the isentropic flow of a perfect gas, and the design constraint is that the nozzle has a specified length.

The nozzle geometry and flowfield are illustrated in Fig. 1. The maximum thrust contour is given by the second-degree polynomial

$$y(x) = a + bx + cx^2 \qquad (1)$$

Specifying the three coefficients a, b, and c uniquely defines the nozzle contour. These coefficients are determined by specifying the attachment angle θ_a, the exit radius y_e, and by requiring that the polynomial contour attach continuously with continuous wall slope to the circular arc initial-expansion contour. Consequently, θ_a and y_e determine a unique nozzle configuration. The optimization procedure searches through the allowable ranges of these two parameters to determine the particular set of values that yields the maximum nozzle thrust. The flowfield and nozzle thrust are calculated by a modified version of the method of characteristics program developed by Pasley and Hoffman.[4] The thrust is the sum of the pressure forces and momentum flux across the throat and the integral of the pressure forces along the nozzle wall.

Direct optimization procedures may be divided into two broad categories: derivative-free methods and derivative methods. Derivative-free methods employ systematic searches through the domain of the objective function (i.e., the thrust) without employing any information about derivatives of the objective function. This class of methods is most useful when the objective function is a function of many variables. Derivative methods employ first derivatives to determine the search direction, and in some cases second derivatives to determine the step size. This class of methods is most useful when the objective function is smooth and depends on only a few variables. This class of methods was used in the present investigation.

The general philosophy of a direct optimization method is to begin with an arbitrary starting point (base point). The optimization method then selects a search direction and a step size for moving the base point toward the optimum. The number of function evaluations required for each base point move and the number of base point moves determine the efficiency of the method, since a function evaluation requires much more computational effort than the optimization logic.

Three direct optimization methods were considered in this study: multidimensional line search, the method of steepest descent, and Newton's method. Multidimensional line search performs a one-dimensional line search in each of the independent variables. A cycle is complete when all of the independent variables have been considered. The method of steepest descent searches along the direction of the gradient of the objective function. Newton's method used both first and second derivatives to determine both the search direction and the step size.

The calculus of variations is an indirect optimization procedure. No constraint is placed on the form of the nozzle contour, $y = y_w(x)$, other than it must be a streamline. Thus, the calculus of variations method yields the absolute best

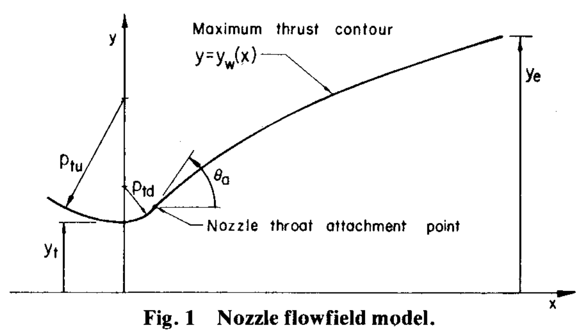

Fig. 1 Nozzle flowfield model.

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Table 1 Comparison of the performance of maximum thrust nozzle contours

Nozzle length, cm	Ambient pressure, atm	Thrust, N Rao nozzle	Thrust, N Polynomial nozzle[a]	ΔF, %
5.897	0	21362.4	21341.9	−0.10
7.799	0	22157.3	22146.6	−0.05
9.704	0	22793.8	22777.8	−0.07
11.616	0	23315.1	23284.8	−0.13
15.993	0	24166.4	24146.9	−0.08
20.259	0	24773.6	24736.2	−0.15
24.518	0	25217.5	25186.8	−0.12
32.375	0	25828.2	25803.7	−0.09
42.968	0	26402.0	26361.5	−0.15
52.153	0	26752.1	26707.2	−0.19
62.221	0	27050.1	26991.4	−0.21
7.331	0.340	21708.0	21565.2	−0.66
20.282	0.340	23626.9	23574.8	−0.22
30.063	0.340	23925.8	23874.2	−0.22
40.892	0.340	23996.5	23919.6	−0.32

[a]Newton's method.

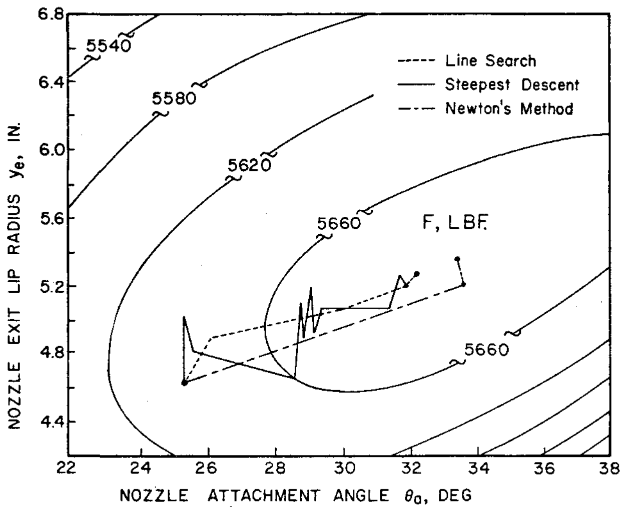

Fig. 2 Example of nozzle optimization.

solution to the specific design problem. The design procedure developed by Rao[1] is a one-dimensional variational problem. The results presented in this paper are compared with results based on Rao's method.

The three direct optimization methods were applied to design a series of maximum thrust nozzle contours for different lengths for two values of the ambient pressure. All operating conditions were typical of modern rocket motor technology. Results are presented to demonstrate the behavior of the direct optimization methods and to show that the thrusts obtained by the second-degree polynomial contours are comparable to the thrusts obtained by calculus of variations contours. The results for a nozzle length of 24.518 cm, illustrated on the surface $F=F(\theta_a,y_e)$, are presented in Fig. 2. The paths to the maximum thrust contour followed by each of the three direct optimization methods are illustrated.

From numerous studies, such as the one illustrated in Fig. 2, the following conclusions regarding the direct optimization methods are drawn: 1) each method converges to the unique maximum thrust contour; 2) the solution obtained is the global maximum; and, 3) each method converges to the global maximum efficiently, without an excessive number of base point moves.

The thrusts developed by Rao contours are compared with the thrusts developed by polynomial contours in Table 1. Both methods predict essentially the same maximum thrust. Some of the differences between the maximum thrust values predicted by the two approaches may exist because the numerical results were obtained with different, although similar, flowfield analysis programs, and the direct optimizations were terminated by nonzero relative tolerances. However, it is likely that the bulk of the differences are due to the restriction of a second-degree polynomial wall contour. Overall agreement between the two approaches is very good. For zero ambient pressure, agreement for all cases is within 0.2%, which is the approximate precision of the method of characteristic flowfield calculation algorithm. The results for an ambient pressure of 0.340 atm do not agree as well, suggesting that a contour with more degrees of freedom may be desirable for this ambient pressure. Thus, polynomial contours effectively approximate maximum thrust nozzle contours, justifying this approach. Higher-degree polynomials could produce even closer agreement between the two procedures, and may be required for other nozzle operating conditions and ambient pressures.

Acknowledgments

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